LECTURE NO 4

- Coordinate system
- Need of transformation
- Transformation from cartesian to cylindrical

Coordinate System & Transformation

- ***EM are functions of space and time.***
- ***The spatial variations of the quantities, define all points uniquely in space in a suitable manner.***
- ***This requires using an appropriate coordinate system".***

- ***An orthogonal system is one in which the coordinates are mutually perpendicular.***
- ***Three best-known coordinate systems:***
- ***The Cartesian***
- ***The circular cylindrical.***
- ***The spherical***

Choice is based on symmetry of problem

CARTESIAN COORDINATES (X, Y, Z)

Point *P can be represented as (x, y, z)*

$$-\infty < x < \infty$$

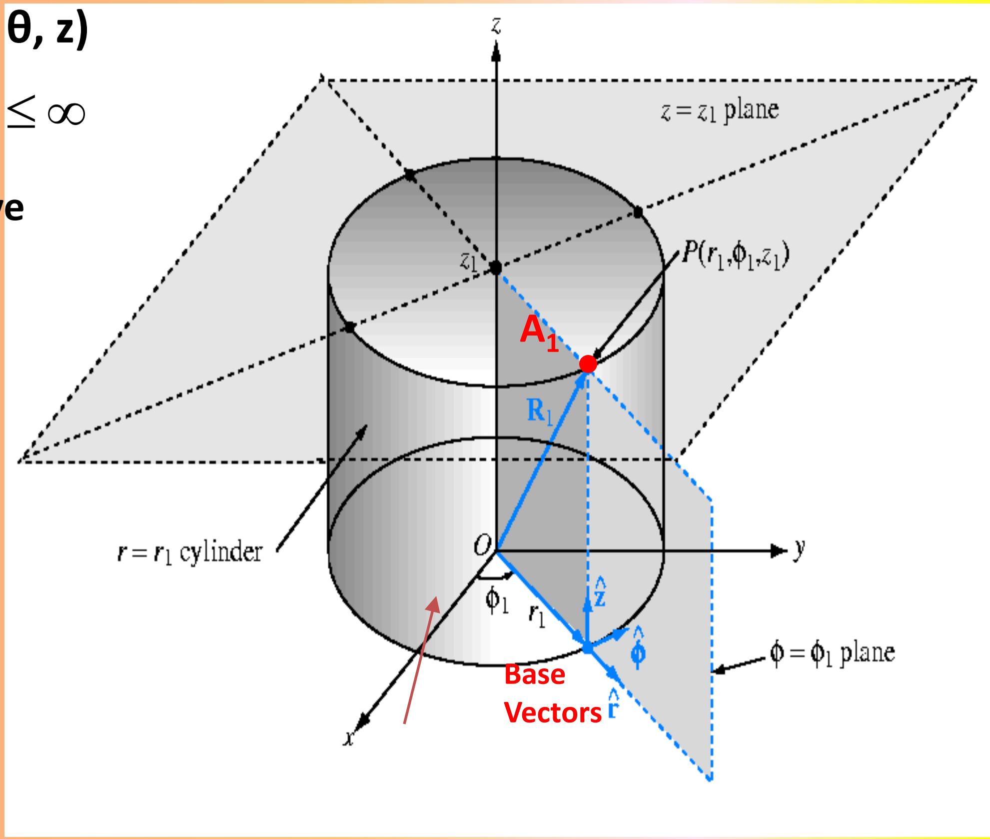
$$-\infty < y < \infty$$

$$-\infty < z < \infty$$

Cylindrical Coordinates

(r, θ, z)

- **r** radial distance in x-y plane $0 \le r \le \infty$
- **Φ** azimuth angle measured from the positive x-axis $0 \le \Phi < 2\pi$
- **z** $-\infty < z < \infty$



Vector representation

$$\vec{A} = \hat{a}\left|\vec{A}\right| = \hat{r}A_r + \hat{\Phi}A_\Phi + \hat{z}A_z$$

Magnitude of A

$$\left|\vec{A}\right| = +\sqrt{\vec{A}\cdot\vec{A}} = +\sqrt{A_r^2 + A_\Phi^2 + A_z^2}$$

Position vector A

$$\hat{r}r_1 + \hat{z}z_1$$

Base vector properties

$$\hat{r}\times\hat{\Phi} = \hat{z},$$

$$\hat{\Phi}\times\hat{z} = \hat{r},$$

$$\hat{z}\times\hat{r} = \hat{\Phi}$$

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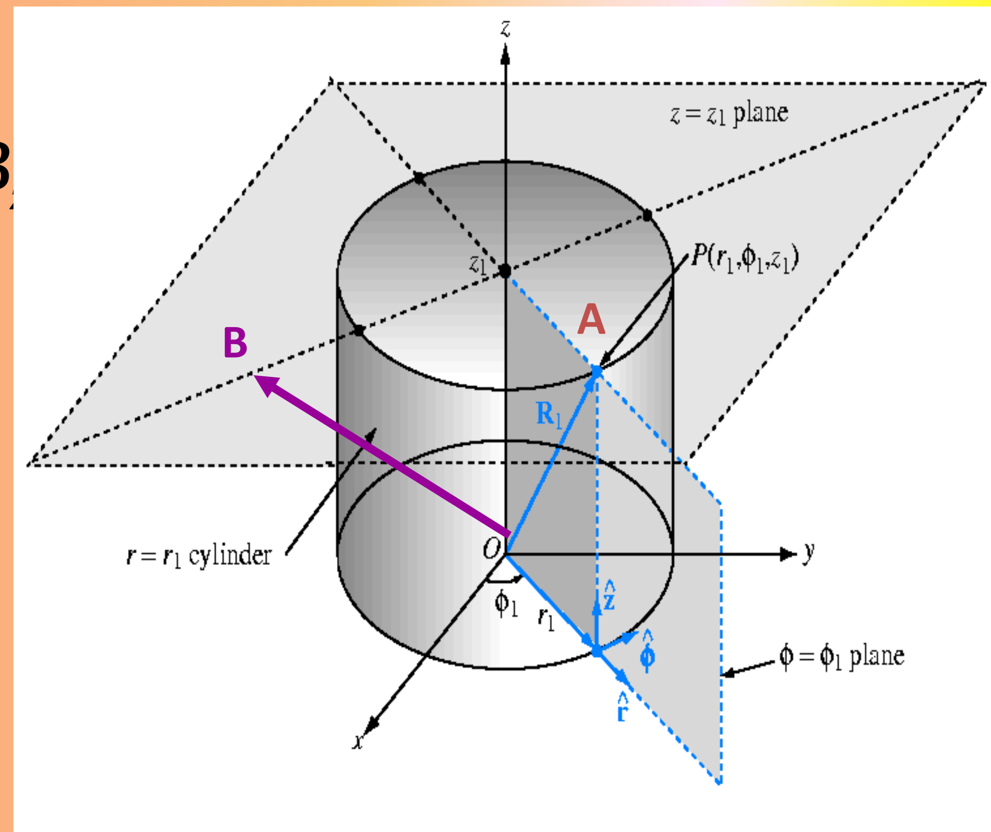
Cylindrical Coordinates

Dot product:

$$\vec{A}\cdot\vec{B} = A_r B_r + A_\phi B_\phi + A_z B_z$$



Cross product:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$$

Spherical Coordinates

(R, θ, Φ)

Vector representation

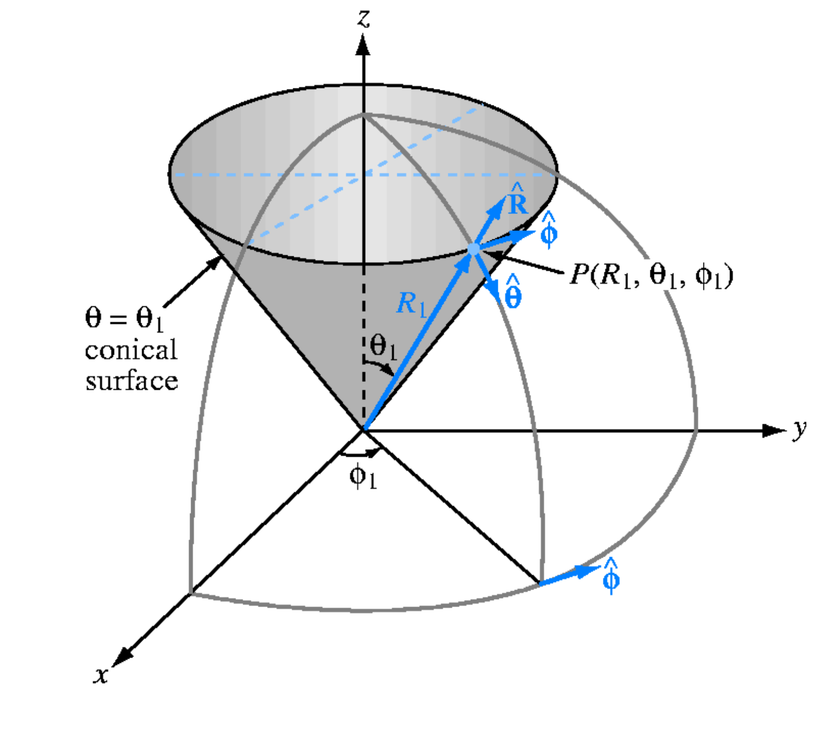
$$\vec{A} = \hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$$

Magnitude of A

$$\left|\vec{A}\right| = \sqrt[+]{\vec{A}\cdot\vec{A}} = \sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$$

Position vector A

$$\hat{R}R_1$$



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Base vector properties

$$\hat{R}\times\hat{\Theta} = \hat{\Phi}, \quad \hat{\Theta}\times\hat{\Phi} = \hat{R}, \quad \hat{\Phi}\times\hat{R} = \hat{\Theta}$$

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Cartesian to Cylindrical Transformation

$$A_r = A_x \cos\phi + A_y \sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

$$A_z = A_z$$

$$r = \sqrt[+]{x^2 + y^2}$$

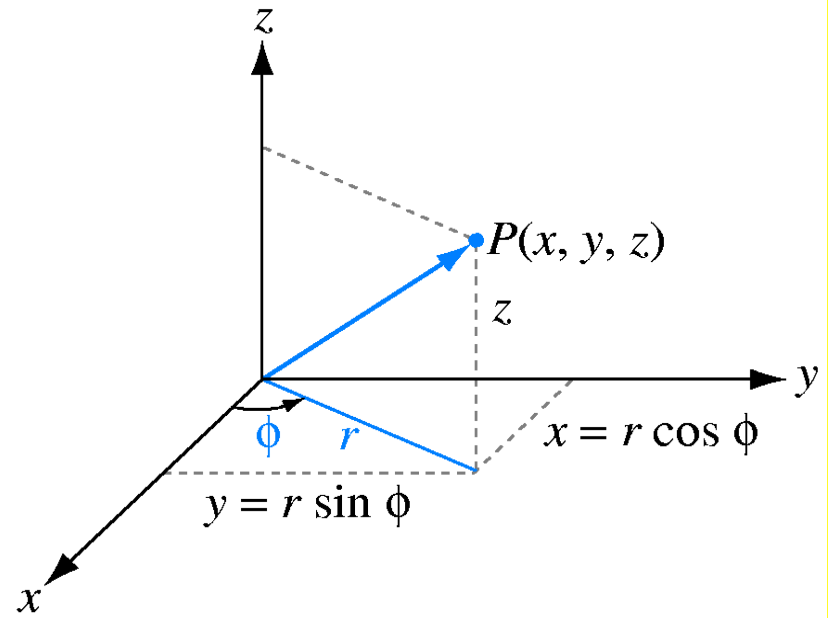
$$\phi = \tan^{-1}(y/x)$$

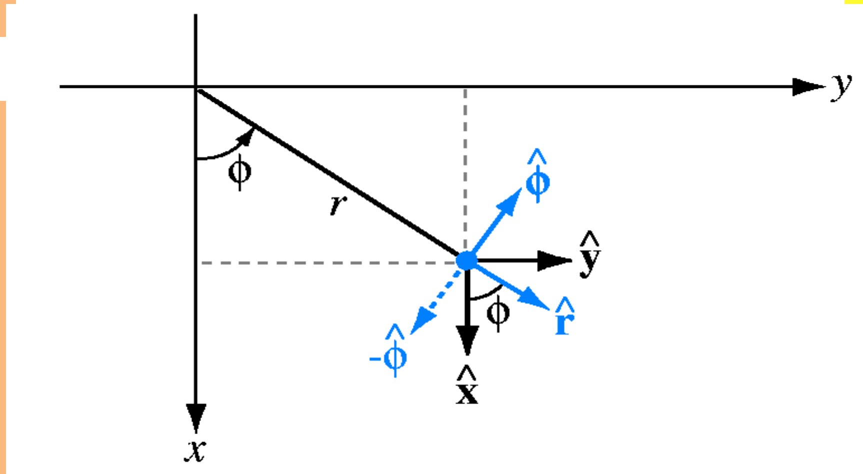
$$z = z$$

$$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$$

$$\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

$$\hat{z} = \hat{z}$$





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CONVERSION OF CARTESIAN VECTOR TO CYLINDRICAL VECTOR

$$\mathbf{a}_x = \cos\phi\,\mathbf{a}_\rho - \sin\phi\,\mathbf{a}_\phi$$

$$\mathbf{a}_y = \sin\phi\,\mathbf{a}_\rho + \cos\phi\,\mathbf{a}_\phi$$

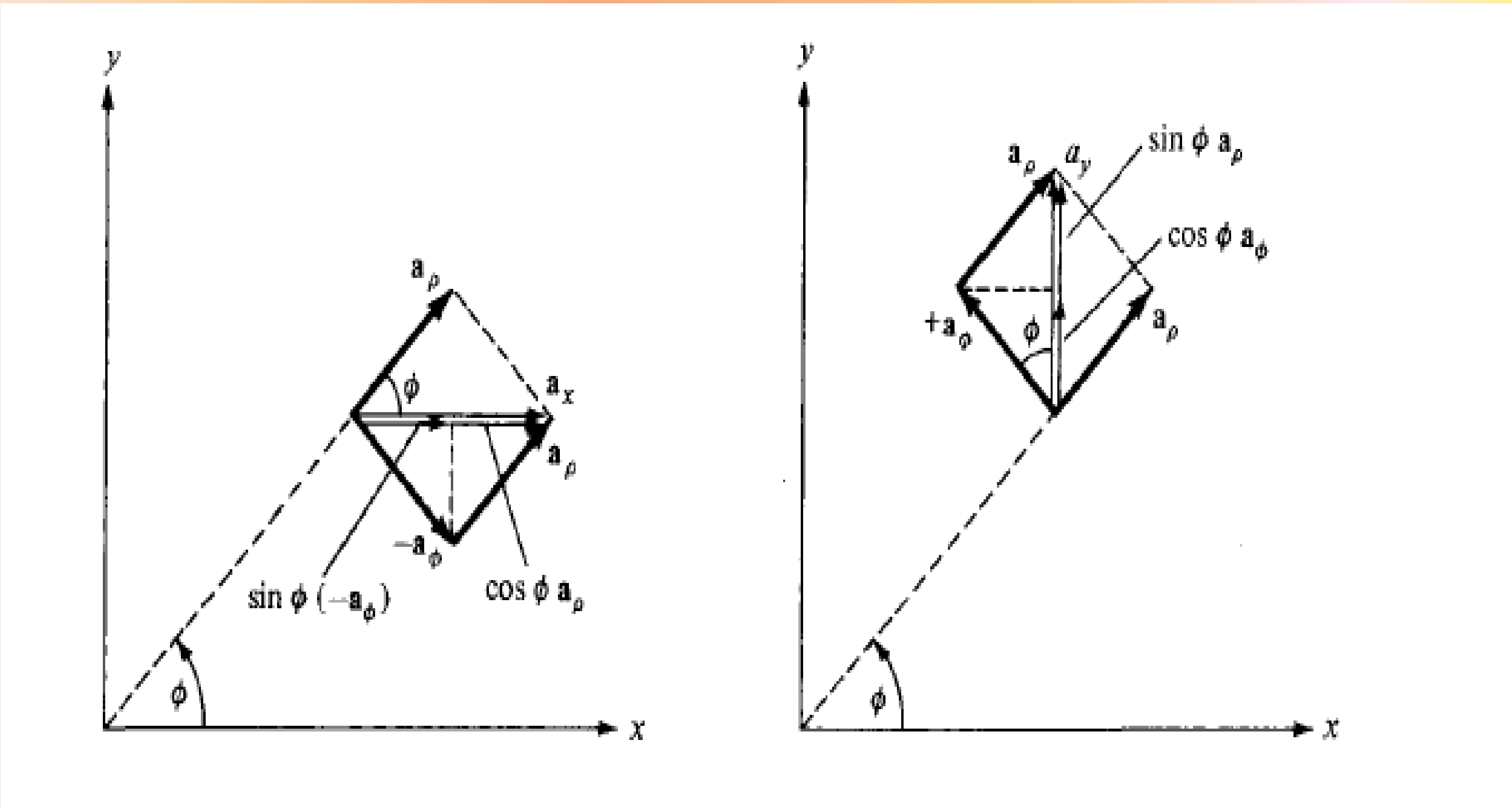
$$\mathbf{a}_z = \mathbf{a}_z$$

$$\mathbf{a}_\rho = \cos\phi\,\mathbf{a}_x + \sin\phi\,\mathbf{a}_y$$

$$\mathbf{a}_\phi = -\sin\phi\,\mathbf{a}_x + \cos\phi\,\mathbf{a}_y$$

$$\mathbf{a}_z = \mathbf{a}_z$$

CONVERSION OF CARTESIAN VECTOR TO CYLINDRICAL VECTOR



CONVERSION OF CARTESIAN VECTOR TO CYLINDRICAL VECTOR

$$A_\rho = A_x \cos\phi + A_y \sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

$$A_z = A_z$$

CONVERSION OF CARTESIAN VECTOR TO CYLINDRICAL VECTOR

In matrix form, we have the transformation of vector **A** from (A_x, A_y, A_z) to (A_ρ, A_ϕ, A_z) as

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

CONVERSION OF CARTESIAN VECTOR TO CYLINDRICAL VECTOR

The inverse of the transformation $(A_\rho, A_\phi, A_z) \to (A_x, A_y, A_z)$ is obtained as

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Spherical Coordinate system

$$0 \leq r < \infty$$

$$0 \leq \theta \leq \pi$$

$$0 \leq \phi < 2\pi$$

$$x = r \sin\theta \cos\phi, \qquad y = r \sin\theta \sin\phi, \qquad z = r \cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad \theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z}, \qquad \phi = \tan^{-1} \frac{y}{x}$$

Spherical Coordinate system

The unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ and $\mathbf{a}_r$, $\mathbf{a}_\theta$, $\mathbf{a}_\phi$ are related as follows:

$$\mathbf{a}_x = \sin\theta\cos\phi\,\mathbf{a}_r + \cos\theta\cos\phi\,\mathbf{a}_\theta - \sin\phi\,\mathbf{a}_\phi$$

$$\mathbf{a}_y = \sin\theta\sin\phi\,\mathbf{a}_r + \cos\theta\sin\phi\,\mathbf{a}_\theta + \cos\phi\,\mathbf{a}_\phi$$

$$\mathbf{a}_z = \cos\theta\,\mathbf{a}_r - \sin\theta\,\mathbf{a}_\theta$$

$$\mathbf{a}_r = \sin\theta\cos\phi\,\mathbf{a}_x + \sin\theta\sin\phi\,\mathbf{a}_y + \cos\theta\,\mathbf{a}_z$$

$$\mathbf{a}_\theta = \cos\theta\cos\phi\,\mathbf{a}_x + \cos\theta\sin\phi\,\mathbf{a}_y - \sin\theta\,\mathbf{a}_z$$

$$\mathbf{a}_\phi = -\sin\phi\,\mathbf{a}_x + \cos\phi\,\mathbf{a}_y$$

Conversion of Cartesian to Spherical Coordinate system and vice-versa

In matrix form, the $(A_x, A_y, A_z) \to (A_r, A_\theta, A_\phi)$ vector transformation is performed according to

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$